Use of statistical interference for tolerance determination in dose measurements

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OBJECTIVES

The most important dosimetry quantity in radiotherapy is absorbed dose to water. Fixed tolerances for absorbed doses under reference conditions measured with ionisation chamber for photon and electron beams are usually 2% and 3% respectively. The aim was to develop a new approach to the evaluation of agreement between measured and reported values based on statistical interference rather than to use fixed tolerance levels.

METHODS

The approach takes partial uncertainties of compared doses into consideration. Three statistical models were proposed describing three different situations to assess the critical values for rejecting the zero hypothesis on agreement:

1) Two measurements performed in time with the same dosimetry chain,

2) Two measurements performed with two different dosimetry chains calibrated at two different calibration laboratories (dose calculations are completely independent),

3) Two measurements performed with two different dosimetry chains which were calibrated at the same calibration laboratory.

For electron dosimetry, the uncertainty of percentage depth dose (PDD) was comprehensively evaluated and its influence on recalculated dose values from reference depth to the depth of maximum dose was taken into account. In the contribution, uncertainties from TRS 398 [1] were taken as an example.

EXAMPLE

Absorbed dose to water formalism for photon beams determined by two different dosimetry chains:

$$D_1 = M_1 N_1 k_{QQ0} \prod_{i=1}^{n} k_{1i} \qquad D_2 = M_2 N_2 k_{QQ0} \prod_{i=1}^{n} k_{2i}$$

μ_{Dj} are mean values of quantity D_j

σ^2_{Dj} are variances of quantity D_j

M_j are independent quantities (i.e. charges) with mean values μ_{Mj} and variances σ^2_{Mj}.

N_j are random quantities (i.e. calibration coefficiens) with mean values σ^2_{Nj} and variances μ_{Nj}.

k_{QQ0} is random quantity with mean value μ_{kQ} and variance σ^2_{kQ}.

$k_{ji}, j = 1, 2,\ i = 1, \ldots, n$ are independent random quantities with mean values μ_{kji} and variances σ^2_{kji}.

PDD for recalculation from reference depth to the depth of dose maximum for absorbed dose to water for electron beams

$$PDD = \frac{g(z_{REF}) s_{w,air}(z_{REF})}{\max_x g(x) s_{w,air}(x)}$$

g values are raw ionisation data

ξ is the uncertainty of measured PDD curve

δ is the uncertainty of depth for PDD curve

ε_x is the uncertainty of measurement at the depth *x*

γ_x is the uncertainty of $s_{w,air}$ at the depth *x*

$$\widehat{PDD} = \frac{\left(g(z_{REF}+\delta)+\xi+\varepsilon_{z_{REF}}\right)\left(s_{w,air}(z_{REF})\gamma_{z_{REF}}\right)}{\max_{x\in\{x_1,\ldots,x_n\}}\left(g(x+\delta)+\xi+\varepsilon_x\right)\left(s_{w,air}(x)\gamma_x\right)}$$

RESULTS

Hypothesis:

$$H_0 : \mu_{D1} = \mu_{D2}$$
$$H_1 : \mu_{D1} \neq \mu_{D2}$$

Test statistics:

$$T = \frac{D_1 - D_2}{\sigma}$$
$$p = 2(1 - \Phi(|T|))$$

$$|D_1 - D_2| > q_{1-\alpha/2}\sigma$$
$$U_{rel}(\alpha) = q_{1-\alpha/2}\frac{\sigma}{\mu_D}$$
$$\frac{|D_1 - D_2|}{\mu_D} > U_{rel}(\alpha)$$

1) $N = N_1 = N_2,\ \mu_N = \mu_{N1} = \mu_{N2},\ \sigma_N = \sigma_{N1} = \sigma_{N2}$

$$\sigma = \sqrt{(\sigma_1^2 + \sigma_2^2)(\sigma_A^2 + \mu_A^2)}$$

Index A relates to $N k_{QQ0}$ which is the same for both dose calculations

$$\mu_A = \mu_N \mu_{kQ},$$
$$\sigma_A^2 = (\sigma_N^2 + \mu_N^2)(\sigma_{kQ}^2 + \mu_{kQ}^2) - \mu_N^2 \mu_{kQ}^2,$$
$$\sigma_j^2 = (\sigma_{Mj}^2 + \mu_{Mj}^2) \prod_{i-1}^{n} (\sigma_{kji}^2 + \mu_{kji}^2) - \mu_{Mj}^2 \prod_{i=1}^{n} \mu_{kji}^2$$
$$U_{rel} = q_{1-\alpha/2}\sqrt{(\xi_1^2 + \xi_2^2)(1 + \xi_A^2)}$$

ξ_A is the coefficient of variation of $N k_{QQ0}$

ξ_j is the coefficient of variation of $M_j \prod_{i=1}^{n} k_{ji}$

2)

$$\sigma = \sqrt{(\sigma_1^2 + \sigma_2^2)(\sigma_{kQ}^2 + \mu_{kQ}^2)}$$

σ_j^2 is variance of $M_j N_j \prod_{i=1}^{n} k_{ji}$ with coefficient of variation ξ_j

$$\sigma_j^2 = (\sigma_{Mj}^2 + \mu_{Mj}^2)(\sigma_{Nj}^2 + \mu_{Nj}^2) \prod_{i=1}^{n} (\sigma_{kji}^2 + \mu_{kji}^2) - \mu_{Mj}^2 \mu_{Nj}^2 \prod_{i=1}^{n} \mu_{kji}^2$$
$$U_{rel} = q_{1-\alpha/2}\sqrt{(\xi_1^2 + \xi_2^2)\left(1 + \left(\frac{\sigma_{kQ}}{\mu_{kQ}}\right)^2\right)}$$

3) „Something in between 1) and 2)“

$$\sigma = \sqrt{\delta^2(\sigma_{kQ}^2 + \mu_{kQ}^2)} \qquad \delta^2 = \delta_1^2 + \delta_2^2 - 2\sigma_L^2 \mu_{M1}\mu_{M2} \prod_{i=1}^{n} \mu_{k1i}\mu_{k2i}$$

CONCLUSIONS

Critical values U_{rel} for photon and electron beams case 1 and 2 were 2.5% and 3.1% (k=2) respectively for typical uncertainties in [1]. When uncertainty of 0.5% (k=1) for electron PDD at z_{ref} was taken into account, critical values for electron beams case 1 and 2 were 2.9% and 3.4% respectively (k=2). The models can be adopted to implement statistical interference into the decision on agreement for all quantities, where partial uncertainties are known.

References

[1] INTERNATIONAL ATOMIC ENERGY AGENCY, Absorbed Dose Determination in External Beam Radiotherapy, Technical Reports Series No. 398, IAEA, Vienna (2000).

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